



Investigation of the dynamics of the vibrating system of a moving vehicle

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Abstract. The paper presents a study of the dynamics of vibrations of the GAZ A31R32 car due to the effects of road bumps. The dynamics of the car's oscillation from the effects of road bumps is an urgent problem, as it has a significant impact on its smoothness, handling, and stability. To study the dynamics of a moving car, a mathematical model has been developed that makes it possible to evaluate the elastic damping properties of a suspension with constant, variable and adjustable parameters. It is a four-bearing, five-mass dynamic model adapted to study roll fluctuations, pitch, and vertical movement of the center of gravity during braking and acceleration. The results of modeling the car's oscillation system show that the elastic-damping properties of the suspension effectively ensure an acceptable level of smoothness during several periods of oscillation, but at the initial moment of time, quite significant amplitudes are observed, which negatively affect the smoothness. This is caused by the impact of road bumps on the car's vibrations and confirms the need to improve the suspension.

Keywords: vibrating system, dynamics, roll, pitch, vehicle body, directional stability, driving safety

Introduction

The article examines the dynamics of the oscillating system of a GAZ 31R32 car (with an elongated base, single-row cab, rear-wheel drive, 4x2 wheel formula and a gross weight of 3,500 kg) with a given profile of road bumps and design parameters of its chassis.

The aim of the work is to study the dynamics of the oscillatory system of the GAZ 31R32 car (hereinafter referred to as the car) when it is moving, taking into account changes in the stiffness characteristics of the front and rear suspensions depending on the operating conditions of the car, as well as to substantiate a constructive solution – installing a less rigid transverse stabilizer on the rear axle – aimed at improving smoothness on uneven roads in urban conditions. operation.

The smoothness and controllability of a car are directly determined by the characteristics of its suspension; therefore, the study of the dynamics of the oscillatory system under the influence of specified road profiles remains one of the key tasks in the design and improvement of vehicles. The vibrations of the body and the elastic suspension elements that occur when the wheels hit the roughness of the road surface affect not only passenger comfort, but also the stability of the wheels' grip on the road, and therefore traffic safety, especially at high speeds and when cornering.

The mathematical model of the dynamics of an oscillatory system is described by a system of second-order differential equations that take into account the inertial and elastic forces acting on the body and wheels when driving over rough roads. The model allows us to study the reaction to vibration effects of a suspension with constant, variable and adjustable parameters. In addition, the model takes into account the influence of unsprung masses and tire elasticity on the stability and handling of the car.

The calculations of the dynamics of the oscillating system have shown that the suspension of the design car in steady state effectively suppresses body vibrations; the amplitude of vibrations of the sprung mass decreases to values that ensure an acceptable level of smoothness after several periods of oscillation. A comparative analysis of the oscillation dynamics calculations has shown that the suspension design provides the most optimal vibration-damping mode, in which the stiffness of the main and additional elastic elements is the same.

In addition, the installation of a rear stabilizer of the required rigidity in the general elastic damping suspension system changes the total angular stiffness of the rear axle, which allows for a more even distribution of vibration damping work between the main elastic elements and the additional stabilizing element. This helps to reduce the amplitude of vibrations at the initial stage after hitting the uneven road surfaces.

The methodology

Analysis of the GAZ 31R32 car vibrating system

It is known that a car interacts with the road through the wheel system [1-7], and with the wheels through the suspension system [8-12]. To study the oscillatory system, including the sprung and unsprung masses of the vehicle, an equivalent suspension design was developed. The front suspension is an independent double-wishbone, while the rear suspension of this vehicle is dependent and uses leaf springs. The suspension system has three main components: wheels and the body. A complete design diagram of the oscillating masses is provided, which directly affect controllability and stability (Fig.1).

The front suspension (independent double wishbone) and rear suspension (dependent leaf spring) are shown separately in the diagram. This diagram serves as the basis for developing a model that includes differential equations for the oscillations of the vehicle's sprung and unsprung

masses and an analysis of the impact of suspension parameters on ride quality, handling, and stability.

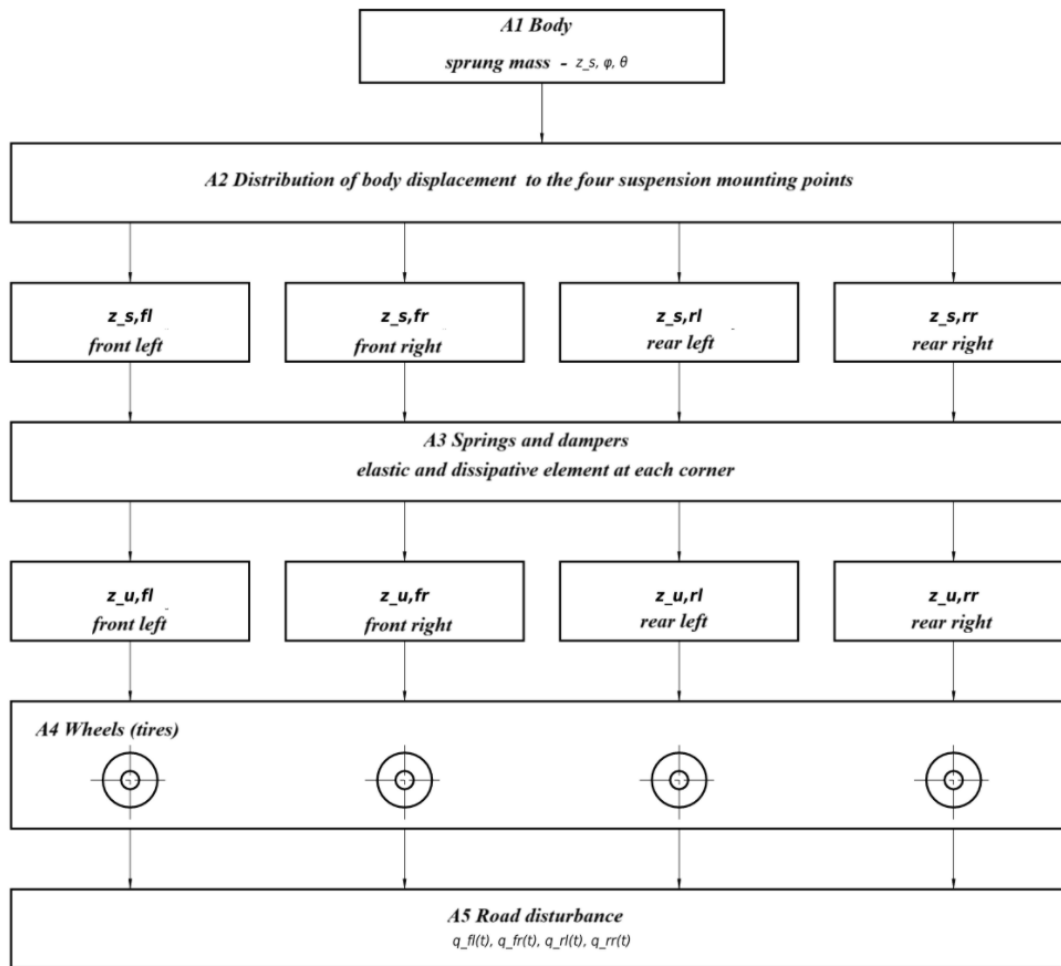


Figure 1. Vibrating system of the GAZ A31R32 (4x2) vehicle

The rigidity characteristics of the tires and body suspension determine the driving performance of the car (Table 1-2).

Table 1. Vehicle component characteristics

Characteristic	Front	Rear
Type	Independent	Dependent: both wheels on the same beam
Suspension scheme	Two levers: upper and lower	On the springs
Elastic element	Spring + shock absorber	Leaf spring
Wheel kinematics	Independent running of each wheel	A joint move
Appointment	manageability	High load capacity

Mathematical model of a gas-filled hydraulic shock absorber. The shock absorber creates a force proportional to the speed of the relative movement between the two nodes:

$$F_{dam} = c \cdot (x_{p1} - x_{p2})$$

c – damping coefficient.

Table 2. Initial parameters of the GAZ31R32 vehicle

Name	Expression	Description
$m_{vehicle\ body}, kg$	990	Mass of vehicle body
m_{GN}, kg	3500	Gross weight GAZ A31R32
$m_{kerb\ weight}, kg$	2510	Curb weight
$I_{roll}, kg \cdot m^2$	378	Interia around roll axis (X)
$I_{pitch}, kg \cdot m^2$	3170	Interia around pitch axis (Y)
m_{wheel}, kg	45	Mass of one wheel with tyre
m_p, kg	80	Mass of driver
$k_{wheel}, N/m$	320000	Tyre radial stiffness
$k_{seat}, N/m$	2200	Seat spring stiffness
$k_{sus_f}, N/m$	42000	Front suspension spring stiffness
$c_{sus_f}, N \cdot s/m$	3500	Front suspension damper viscosity
$k_{sus_r}, N/m$	65000	Rear suspension spring stiffness
$c_{sus_r}, N \cdot s/m$	5000	Rear suspension damper viscosity
$c_{seat}, N \cdot s/m$	700	Seat damper viscosity
r_{wb}, m	3.745	Wheelbase (distance between front and rear axles)
	–	
r_{ftw}, m	1.75	Front track width (centre)
r_{rtw}, m	1.56	Rear track width (centre)
hb, m	0.04	Bump height
wb, m	0.075	Bump width
$v_{veh}, m/s$	11.11	Vehicle speed
tb, s	wb/v_{veh}	travel time
td, s	r_{wb}/v_{veh}	rear wheel delay

Model of the GAZ A31R32 car vibrating system

To solve the problem of dynamics simulating the state of the oscillatory system during braking and acceleration of the car, its dynamic model was developed. The four-bearing, five-mass dynamic model is adapted to study the roll, pitch, and vertical displacement of the center of gravity of the GAZ 31R32 design vehicle during braking and acceleration. At the same time, we consider the car as an oscillatory system consisting of several masses interconnected by elastic damping connections. For a four-foot, five-mass model, the car body is considered as one large springing mass (m_s), which has vertical movement. Each of the four wheels is modeled as a separate unsprung mass ($m_{ufl}, m_{ufr}, m_{url}, m_{urr}$) with its own vertical movements. This model has 7 degrees of freedom: z_s -vertical movement of the body, ϕ – (roll) body roll, θ – (pitch) body pitch, $z_{ufl}, z_{ufr}, z_{url}, z_{urr}$ – vertical movement of the wheels of the front left and right, rear left and right wheels, respectively.

The mathematical model of the system dynamics is written as follows [6-10]:

The equations of dynamics of wheel oscillation:

Front left wheel:

$$m_{wh,fl}\ddot{z}_{u,fl} = k_{susfl}(z_{s,fl} - z_{u,fl}) + c_{susfl}(\dot{z}_{s,fl} - \dot{z}_{u,fl}) - k_{tfl}(z_{u,fl} - z_{0,fl}) \quad (1)$$

Front right wheel

$$m_{wh,fr}\ddot{z}_{u,fr} = k_{susfr}(z_{s,fr} - z_{u,fr}) + c_{susfr}(\dot{z}_{s,fr} - \dot{z}_{u,fr}) - k_{tfr}(z_{u,fr} - z_{0,fr}) \quad (2)$$

Rear left wheel

$$m_{wh,rl}\ddot{z}_{u,rl} = k_{susrl}(z_{s,rl} - z_{u,rl}) + c_{susrl}(\dot{z}_{s,rl} - \dot{z}_{u,rl}) - k_{trl}(z_{u,rl} - z_{0,rl}) \quad (3)$$

Rear right wheel

$$m_{wh,rr}\ddot{z}_{u,rr} = k_{susrr}(z_{s,rr} - z_{u,rr}) + c_{susrr}(\dot{z}_{s,rr} - \dot{z}_{u,rr}) - k_{trr}(z_{u,rr} - z_{0,rr}) \quad (4)$$

$m_{wh} \cdot \ddot{z}_u = F$ – force acting on an unsprung mass; m_{wh} – the mass of each wheel clamped between two springs; k_{tfl} – front left wheel tire stiffness, z_{0fl} – road profile, k_{susfl} – stiffness of the suspension spring of the front left wheel, $z_{s,fl}$ – vertical movement of the sprung mass (body) above the wheel, $z_{u,fl}$ – vertical movement of the unsprung mass of the front left wheel, c_{susfl} – damping coefficient of the front left wheel shock absorber, $\dot{z}_{s,fl}$ и $\dot{z}_{u,fl}$ – the vertical velocities of the sprung and unsprung masses, respectively.

When a car is moving from the side of the road, a kinematic effect occurs, determined by the profile of the road. The roughness of the road causes the car to wobble. According to the conditions of the task, the road unevenness affects only the left wheels of the car, while the right wheels move on a flat surface. In this regard, the road profiles for each wheel are set by the following dependencies:

$$\begin{aligned} z_{0,fl}(t) &= \frac{hb}{2} + wb(t), & z_{0,fr}(t) &= 0 \\ z_{0,rl}(t) &= \frac{hb}{2} + wb(t), & z_{0,rr}(t) &= 0 \end{aligned}$$

$wb(t)$ – ledge width, hb – height of the projection.

The equation of dynamics of vertical body vibration:

$$\begin{aligned} m_{body}\ddot{z}_s &= -(F_{fl} + F_{fr} + F_{rl} + F_{rr}) \\ F_{fl} &= k_{susfl}(z_{s,fl} - z_{u,fl}) + c_{susfl}(\dot{z}_{s,fl} - \dot{z}_{u,fl}) \\ F_{fr} &= k_{susfr}(z_{s,fr} - z_{u,fr}) + c_{susfr}(\dot{z}_{s,fr} - \dot{z}_{u,fr}) \\ F_{rl} &= k_{susrl}(z_{s,rl} - z_{u,rl}) + c_{susrl}(\dot{z}_{s,rl} - \dot{z}_{u,rl}) \\ F_{rr} &= k_{susrr}(z_{s,rr} - z_{u,rr}) + c_{susrr}(\dot{z}_{s,rr} - \dot{z}_{u,rr}) \end{aligned} \quad (5)$$

$F_{fl}, F_{fr}, F_{rl}, F_{rr}$ – the forces in the suspensions of the front left, front right, rear left and rear right wheels, respectively, transmitted through the suspension elements (spring and shock absorber) between the body (sprung mass) and the unsprung masses of the corresponding wheels.

The roll dynamics equation:

$$I_{Roll}\ddot{\varphi} = -\frac{r_{ftw}}{2}F_{fl} + \frac{r_{ftw}}{2}F_{fr} + \frac{r_{rtw}}{2}F_{rl} + \frac{r_{rtw}}{2}F_{rr} \quad (6)$$

I_{Roll} – moment of inertia relative to the longitudinal axis.

The equation of pitch dynamics:

$$I_{pitch}\ddot{\theta} = -a(F_{fl} + F_{fr}) + b(F_{rl} + F_{rr}) \quad (7)$$

I_{pitch} – moment of inertia relative to the transverse axis, a – distance from the center of mass to the front axle, b – distance from the center of mass to the rear axle.

To calculate the movements of the body at the attachment points of the suspensions in COMSOL Multiphysics version 6.4, establishes kinematic links between the coordinates of the body z_s , φ и θ :

$$z_{s,fl} = z_s + l_f \varphi - a\theta$$

$$z_{s,fr} = z_s - l_f \varphi - a\theta$$

$$z_{s,rl} = z_s + l_r \varphi + b\theta$$

$$z_{s,rr} = z_s - l_r \varphi + b\theta$$

$z_{s,fl}$, $z_{s,fr}$, $z_{s,rl}$, $z_{s,rr}$ – suspension mounting points on the body, z_s – vertical movement of the body's center of mass, φ – roll angle, θ – pitch angle, l_f – half of the front track, l_r – half of the rear track.

The equation of seat swing dynamics

$$m_{p,fl} \ddot{z}_{p,fl} = k_{seat,fl} (z_{s,fl} - z_{p,fl}) + c_{seat,fl} (\dot{z}_{s,fl} - \dot{z}_{p,fl}) \quad (8)$$

$$m_{p,fr} \ddot{z}_{p,fr} = k_{seat,fr} (z_{s,fr} - z_{p,fr}) + c_{seat,fr} (\dot{z}_{s,fr} - \dot{z}_{p,fr}) \quad (9)$$

$$m_{p,rl} \ddot{z}_{p,rl} = k_{seat,rl} (z_{s,rl} - z_{p,rl}) + c_{seat,rl} (\dot{z}_{s,rl} - \dot{z}_{p,rl}) \quad (10)$$

$$m_{p,rr} \ddot{z}_{p,rr} = k_{seat,rr} (z_{s,rr} - z_{p,rr}) + c_{seat,rr} (\dot{z}_{s,rr} - \dot{z}_{p,rr}) \quad (11)$$

Findings/Discussion

Road profile: rectangular wave with a height of 4 cm, a width of 7.5 cm, a speed of 40 km/h (11.11 m/s). The left wheels drive on an uneven road, while the right wheels move on a flat surface. The figure shows that the body rotates around the longitudinal axis (Roll), i.e., the left side of the body is raised. A high Roll value means that only the left wheels are traveling on an uneven road; the load is not symmetrical and, accordingly, the body is tilting. Fig.2 shows the dynamics of the car's roll (blue line), tilt (green), and rise (red) due to the roughness of the road under the left wheels. You can see that with a given roughness of the road, the bank angle is much larger than the tilt angle.

Dynamics of the car's roll speed fluctuation (Roll-Lateral rotation), tilt (Pitch-Longitudinal rotation), and lift (Heave-Vertical displacement) are shown in Fig. 3. The velocity graph shows two different frequencies: low and high, corresponding to the natural frequencies for the system components. At the initial moment of time, the suspension is affected by an external influence from the side of the road profile, which causes sudden fluctuations in the body, and maximum speeds are observed. The maximum speed values are reached at a time of 0.09 s; then, due to the action of the damping elements of the suspension, the vibrations attenuate. The largest amplitude is observed in vertical displacement (the red line is vertical displacement). The graph clearly shows that the roll (Lateral rotation or Roll-blue line) shows a sharp decaying oscillation after reaching the peak value, and the vertical velocity of the center of gravity of the body (Vertical displacement or heave -red line) shows an exponentially decaying character. Consequently, the body makes small vertical fluctuations relative to the equilibrium position. Stability is maintained for longitudinal rotation (Pitch – green line).

The results obtained indicate that, in general, the suspension system of the design car suppresses body vibrations quite effectively, but at the initial moment of time the vibrations have

rather high amplitudes, which primarily affect the smoothness of the car. This circumstance arises due to the influence of the car's suspension parameters.

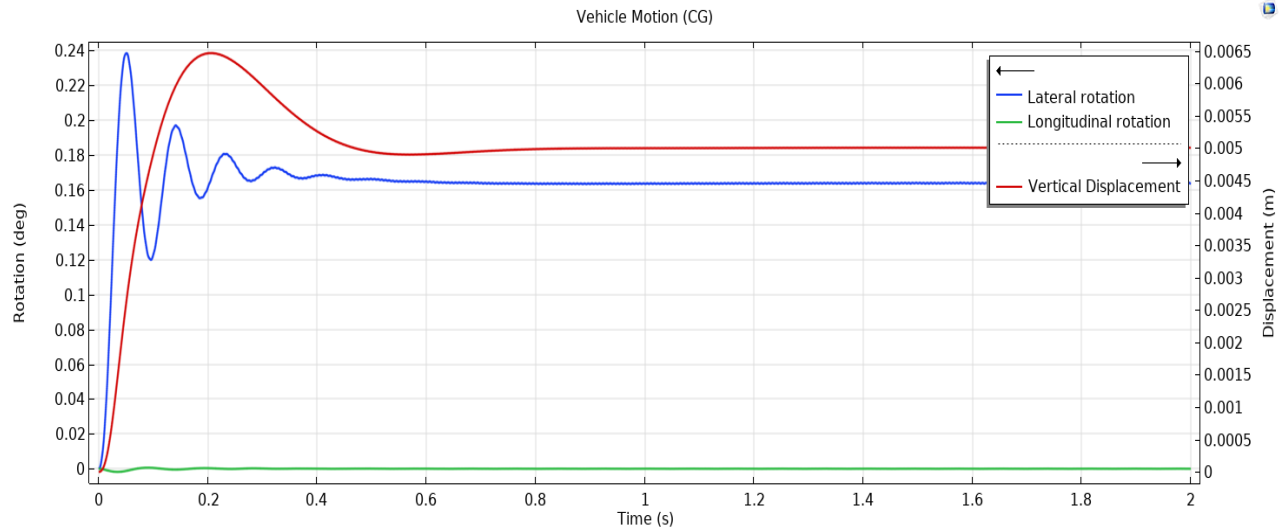


Figure 2. Roll oscillation dynamics (roll-lateral), tilt (pitch-Longitudinal), and lift of the center of gravity (heave-Vertical displacement)

The calculation results show that the suspension of the design vehicle impairs its smoothness due to the effects of road irregularities (longitudinal and transverse) of the road network, which are common in large cities due to excessive traffic intensity. Ultimately, this determines the safety of vehicles. Moreover, it should be noted that in the road and climatic conditions of Kazakhstan, the most important indicator characterizing the safety of both single vehicles and traffic flows is the traction qualities of roads. Therefore, the assessment of vehicle safety is carried out taking into account the above road conditions [13-20].

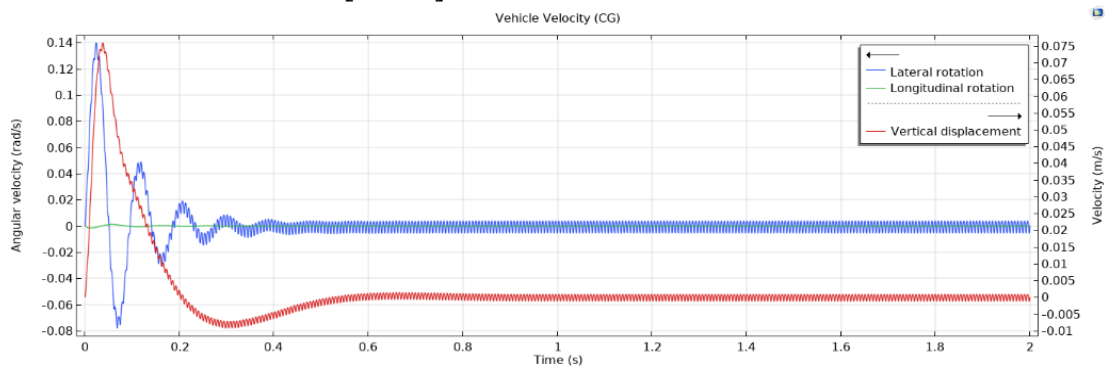


Figure 3. Dynamics of the car's roll speed fluctuation (Roll-Lateral rotation), tilt (Pitch-Longitudinal rotation) and the rise (Heave-Vertical displacement)

To assess the vibration effect on the passenger, graphs are presented in the form of dynamics of vertical movement and acceleration fluctuations in four seat locations (Fig.4-5). An analysis of the time dependence of vertical movements and seat accelerations shows that the maximum displacement at $z_{max} = 0.0129 \text{ m}$ (12.9 mm) is observed due to the impact of road bumps in the seat suspension system (Front-Left and Rear-Left). This indicates that most of the energy was

transferred to the left side of the wheels due to the difference in vertical loads between the sides of the car.

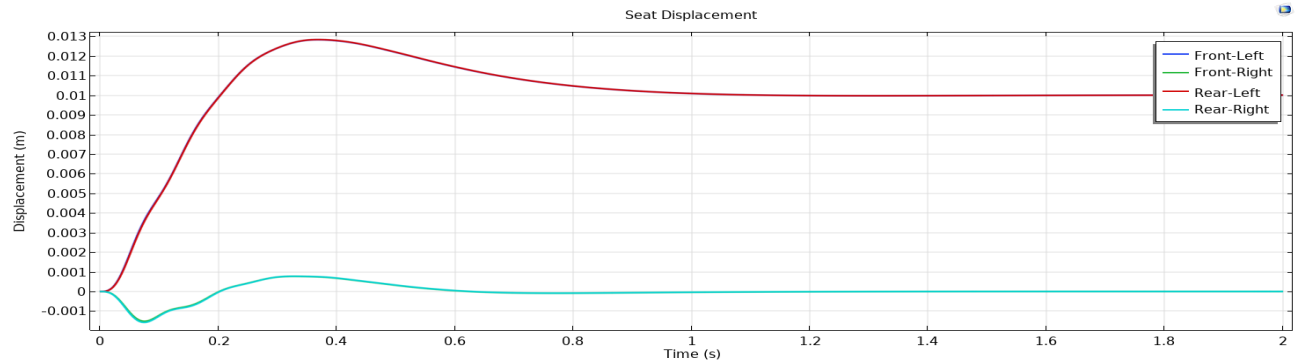


Figure 4. Dynamics of fluctuation of vertical movement and tilt of four seats

The greatest vertical acceleration is recorded synchronously for the rear left and front left (Rear-Left and Front-Left) seats and is approximately 2.5 m/s^2 . The acceleration indicator is the main indicator of comfort, since a person perceives acceleration rather than displacement. The graph shows that with vertical acceleration, $a_{\max} = 2.5 \text{ m/s}^2$ the passenger experiences the greatest impact from road unevenness due to the resulting vibration of the suspension system, which gradually goes out with a corresponding decrease in amplitude. The minimum amplitude values are reached at $a = 0.5 \div 0.75 \text{ m/s}^2$, and at $t \approx 0.6 \text{ s}$ the acceleration goes into harmonic mode. This indicates a fairly effective vibration isolation of the body and an acceptable level of impact on passengers.

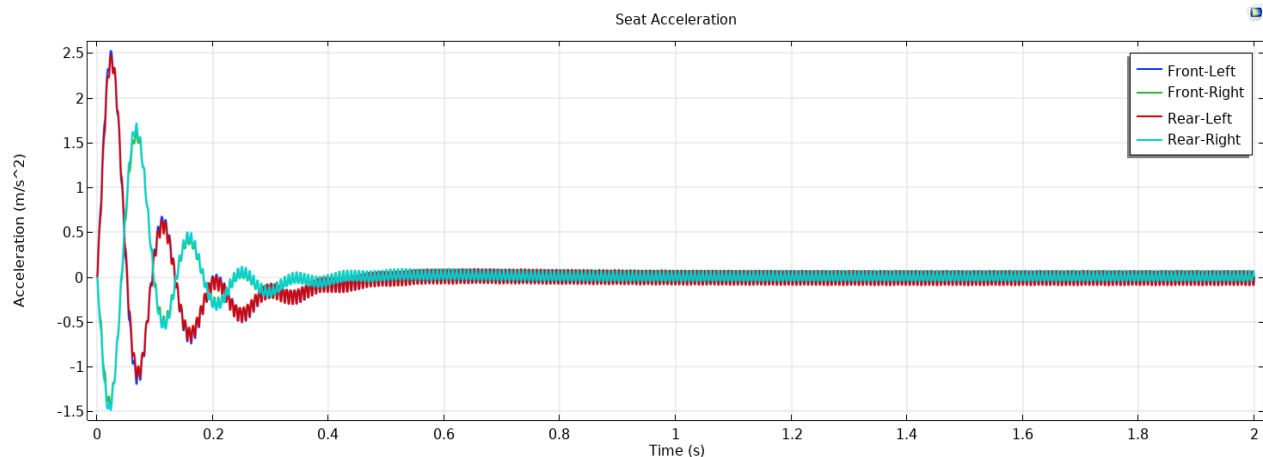


Figure 5. Dynamics of acceleration fluctuations in the rise and tilt of the seat

The above calculation results are consistent with the classical theory of vehicle oscillations and show that when a car wheel encounters a road irregularity, its ride smoothness deteriorates, which is further exacerbated by the standard rear-dependent suspension of the calculated vehicle. At the same time, these impacts of road bumps (longitudinal or transverse) also impair the handling and stability of the car and, respectively, the safety of its movement. Taking into account the propensity of a rear-wheel drive car to oversteer, the rigid dependent suspension of the design vehicle during frequent turns in urban conditions with incomplete loading will tend to skid the rear axle in adverse road conditions, which will also worsen the safety of its movement. These circumstances

seem to be especially important for the road and climatic conditions of our country, where the traction quality of roads periodically tends to decrease.

It is known that an increase in the stiffness of the suspension leads to a decrease in the margin of adhesion of the wheels to the road of the corresponding axle [21]. Therefore, if the front suspension of the calculated car is more rigid relative to the rear, then the balance of its turnability will shift towards insufficient. At the same time, the margin of adhesion of the wheels of the rear axle to the road will increase, which will help to increase the safety of its movement. Therefore, for vehicles primarily used in urban conditions with partial load, it is recommended to provide a less rigid rear suspension.

Thus, taking into account the calculation results and the analysis of the suspension operation, the paper presents a draft stabilizer for the lateral stability of the rear axle of the design vehicle in the form of a drawing of design documentation (рисунке 6).

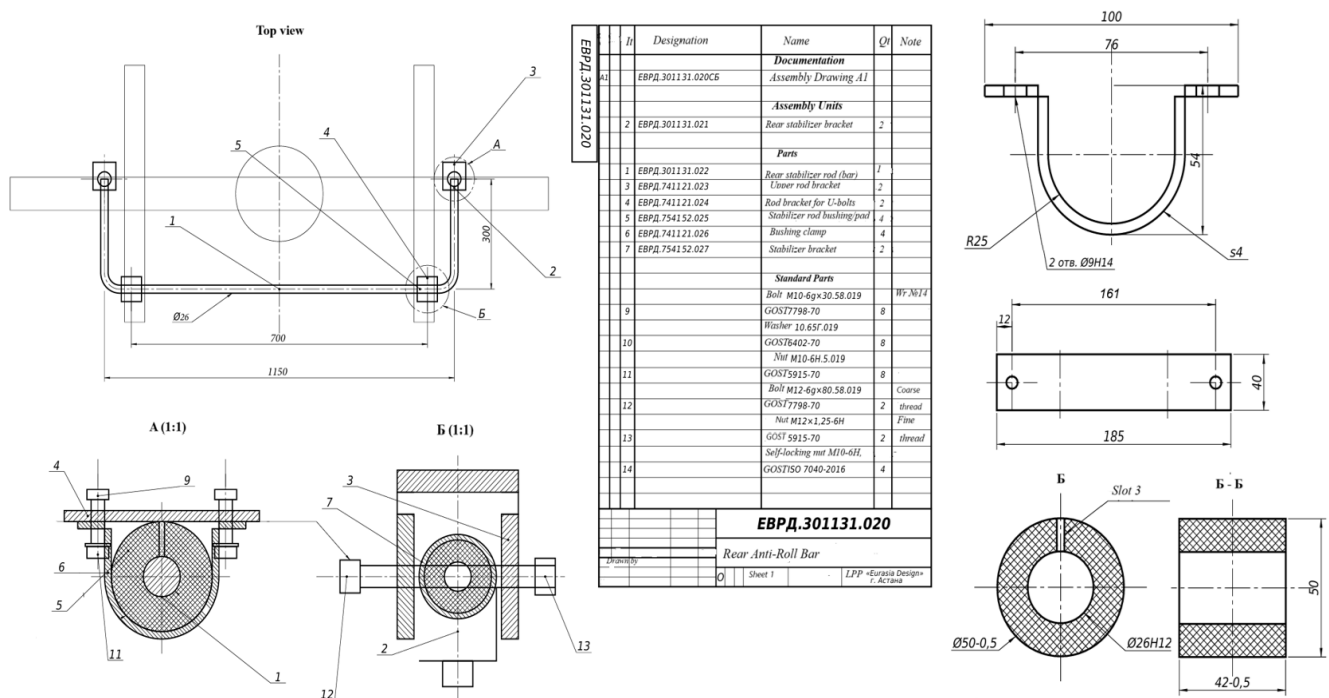


Figure 6. Engineering Drawing of the Rear Anti-Roll Bar

This project will increase the diagonal mobility of the suspension, which will ensure a smoother ride by gently working out holes and road bumps. This is confirmed by the fact that, in relation to the calculated case of hitting the left wheel of the calculated car on an unevenness, less force will be transmitted to the opposite wheel. To do this, it is necessary to equip the rear suspension of the car with a similar lateral stability stabilizer. It is not difficult to install such an option for vehicles mainly intended for frequent use in urban environments, since the disassembly and installation of the stabilizer is of little complexity.

This upgrade will allow the rear suspension of the design vehicle to work more independently by reducing stiffness by about 25%, which will reduce the amplitude and frequency of body vibrations from road bumps in the urban cycle and will increase the reserve of wheel traction when exposed to lateral forces, including cornering. Also, the proposed recommendation will make it possible to safely operate such vehicles with a full load, but on mostly straight tracks.

Conclusion

According to the obtained results of modeling the dynamics of the oscillation, the calculated graphs show that the impact of road unevenness on the suspension of the car at the initial moment causes sharp body vibrations; that is, their maximum values are observed at a time of 0.09 s, which subsequently fade due to the action of damping suspension elements. At the same time, the greatest amplitude of oscillation is observed in vertical movement. From the graphs obtained, the roll fluctuations after reaching the peak value show a sharp attenuation, and the vertical fluctuations in the center of gravity of the body show an exponentially attenuating character. Consequently, after the initial jump of vertical oscillation, the body is relatively quickly restored to the equilibrium position.

At the same time, stability remains constant for the calculated pitch conditions. The results obtained indicate that, in general, the suspension system of the design vehicle suppresses body vibrations quite effectively, but at the initial moment, rather high oscillation amplitudes are demonstrated, which worsens its operational properties. High body vibration amplitudes occur due to the influence of suspension parameters, as it impairs the smoothness of the car under the influence of road bumps, which, along with the traction qualities of the roads, determine its directional stability and traffic safety.

Therefore, to improve the safety of a design vehicle with an incomplete load for operation on the urban road network in the road and climatic conditions of Kazakhstan, it is proposed to install an appropriate lateral stability stabilizer on its rear axle. This recommendation will reduce the amplitude and frequency of vibrations of the car body in the urban cycle of operation and increase the grip margin of the wheels of its rear axle. Therefore, the proposed recommendation makes it possible to improve the smoothness and handling of the car.

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The contribution of the authors:

T.N. Bekenov - Formulation of the research problem, analysis of theoretical data, participation in writing the main text of the article, development of the scientific interpretation of the results, coordination of the joint work of the author team.

Z.K. Sattinova - Collection and analysis of theoretical data, participation in writing the main text of the article and in the analysis of the results obtained, editing of the article.

Zh.T. Nussupbek - Participation in developing the research concept, collection and systematization of data on the vehicle's oscillatory system, participation in writing the main text of the article and in the analysis of the results obtained, scientific interpretation of the results and ensuring scientific style.

Zh.T. Tassybekov - Collection and systematization of data on the suspension system, preparation of graphic materials (diagrams/schemes), participation in discussing the results and ensuring the correctness of the wording.

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